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On the asymmetric response of aquifer water level to floods and droughts in Illinois

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Abstract. Here we analyze observed characteristics of the natural variability in the regional-scale hydrological cycle of Illinois, including the soil and atmospheric branches. This analysis is based on a consistent data set that describes several hydrological variables: the flux of atmospheric water vapor, incoming solar radiation, precipitation, soil moisture content, aquifer water level, and river flow. The climatology of the average regional hydrological cycle has been estimated. Variability in incoming solar radiation, not precipitation, is the main forcing of the seasonal variability in evaporation, soil moisture content, aquifer water level, and river flow. While precipitation plays a minor role in shaping the natural variability in the regional hydrological cycle at the seasonal timescale, variability in precipitation is the major factor in shaping the natural variability in the regional hydrological cycle at the interannual timescale. The anomalies in the different variables of the regional hydrological cycle have been computed and the persistence patterns of extreme floods and droughts have been compared. The 1988 drought left a signature in the aquifer water level that is significantly more persistent than the corresponding signature for the 1993 summer flood. The discharge from unconfined groundwater aquifers to streams (base flow) provides an efficient dissipation mechanism for the wet anomalies in aquifer water level. However, the nonlinear dependence of the groundwater discharge on aquifer water level (groundwater rating curve) may explain why droughts leave a significantly more persistent signature on groundwater hydrology, in comparison to the signature of floods. This nonlinearity has been attributed to the increasing degree by which the unconfined aquifers get connected to the channels network, as the aquifer water level rises leading to higher drainage density. The potential implications of these results regarding the impact on regional water resources due to any future climate change are discussed.

1. Introduction

Understanding the mechanisms of natural variability in any regional hydrological cycle is a necessary step before we can make any credible predictions about the impact of future climate change on hydrology and water resources. In this paper we study direct observations on the regional-scale hydrological cycle of Illinois with the objectives of (1) identifying the statistical patterns and physical mechanisms of natural variability in the regional cycle at the seasonal and interannual timescales and (2) investigating the mechanisms of amplification and dissipation of hydrological anomalies which define how hydrological floods and droughts propagate within the regional cycle from the atmosphere down to the groundwater aquifers, including the response of the regional aquifers to floods and droughts.

The hydrological cycle of North America, including Illinois, has been the subject of several studies: those of *Benton et al.* [1950], *Benton and Estoque* [1954], and *Rasmusson* [1967, 1968, 1971]. At a small scale, several studies were performed on the hydroclimatology of Illinois, mostly by scientists from the Illinois State Water Survey (ISWS). The study of *King* [1984] describes the history of Illinois climate during the last 10,000 years on the basis of pollen data from central Illinois. The more recent climate of Illinois, during the last 100 years, has been analyzed by *Changnon* [1984] on the basis of recorded observations. The relationship between climate variability and droughts occurrence was discussed by *Easterling and Changnon* [1987], and *Changnon* [1987], who defined the climatology of droughts in Illinois. Figure 1, which is taken from *Changnon* [1987], illustrates schematically how precipitation deficiencies during a hypothetical 4-year period are translated in delayed fashion through other components of the hydrological cycle. In this paper we present a graphical illustration of the propagation of hydrological anomalies through the hydrological cycle (similar to Figure 1) that is based on direct observations of the relevant hydrological variables.

The statistical relationship between precipitation and groundwater levels, which is implied by Figure 1, has been studied by *Changnon et al.* [1988]. A significant correlation has been found between precipitation in any month and groundwater level in the following month. Thus the temporal pattern of water level in groundwater aquifers should lag precipitation patterns by about one month. An example for this association is presented by *Olson* [1982], who studied the response of the groundwater system in Illinois to the drought event of 1980–1981. *Changnon et al.* [1988] calibrated a time series model using the data for the period 1960–1979 to predict the groundwater levels during the 1980–1981 drought.

The possible effects on the potential for flooding due to the

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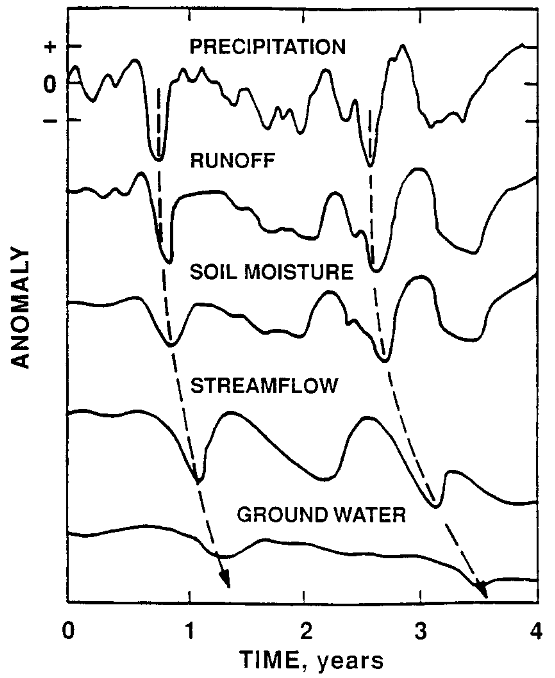

Figure 1. The propagation of precipitation anomaly through the soil branch of the hydrological cycle. (From *Changnon* [1987]).

wetter climate in Illinois since 1940 have been addressed by *Changnon* [1983]. *Kohlhase* [1987] reported a significant rise in groundwater level in the East St. Louis area of Illinois between 1981 and 1985. *Smith and Richman* [1993] investigated the hydrologic and water resource effects induced by a change from a dryer to a wetter climatic regime over Illinois. They have shown evidence of a consistent increase by about 1 m in the seasonal cycle of groundwater level for the period of 1970–1990 compared to that of 1950–1970. *Smith and Richman* [1993] suggest that "this rise in well level may have been climatically driven but comparatively little is known about the response of shallow water table to long-term climate shifts" (p. 260).

In this study we investigate the patterns of hydrological floods and droughts in Illinois as they propagate from the atmosphere into the soil and down to the groundwater aquifers. The climatology of the monthly averages as well as the corresponding anomalies, namely floods and droughts, in several hydrological variables will be studied. These variables include the flux of atmospheric water vapor, precipitation, soil moisture content, groundwater level, and river flow. The response of the aquifer water level to floods and droughts is investigated using direct observations. We propose that aquifers in Illinois respond differently to floods and droughts resulting in amplification (dissipation) of dry (wet) anomalies. The physical mechanisms responsible for these patterns of hydrological anomalies are investigated using analytical techniques.

The paper is organized in seven sections. The following section includes a description of the data sets used in this study. In section 3 we present the climatology of the average regional hydrological cycle in Illinois. Section 4 includes a description of the anomalies in the different hydrological variables and a discussion of how these events propagate between the different reservoirs of the regional hydrological cycle. Section 5 includes a new application of the crossing theory in investigation of the asymmetry in the persistence patterns of floods and droughts in Illinois. In section 6 we propose a physical mechanism for explaining the observed asymmetric response of aquifers in Illinois. This section includes a theoretical analysis that attributes the observed asymmetry in the response of the aquifers to the nonlinearity of the groundwater rating curve (the relationship between base flow and aquifer water level) and a discussion of the physical basis for this nonlinearity. The conclusions of this paper are presented in section 7, which also includes a discussion of the implications of the results from this study regarding the impact of climate change on water resources in Illinois and an outline of related future research.

2. Data

The data set used in this study describes the following variables: atmospheric specific humidity, horizontal wind velocity, incoming solar radiation, precipitation, soil moisture content, groundwater level, and river flow. Figure 2 shows the boundaries of the study area as well as the locations of the measurements stations. The data on specific humidity and on wind is a subset of the National Centers for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) global reanalysis data [*Kalnay et al.*, 1996]. This data set has a 6-hour temporal resolution and a $2.5° \times 2.5°$ horizontal spatial resolution. In this study we use data at the following eight pressure levels: 1000, 925, 850, 700, 600, 500, 400, and 300 mbar. Above 300 mbar the atmospheric water vapor content is negligible. The atmospheric flux of water vapor, as well as atmospheric moisture convergence, were estimated from the data on wind velocity and specific humidity [*Eltahir and Bras*, 1994; *Yeh et al.*, 1998].

The radiation data have been retrieved from the NASA Langley Research Center Global Long-Term Surface Radiation Budget Data Set, which contains global information on both net longwave radiation and net solar radiation. This data set contains monthly averages from the period of July 1983 to June 1991 (i.e., 8 years). They are produced on a global $2.5° \times 2.5°$ grid based on satellite data and cloud parameters from the International Satellite Cloud Climatology Project (ISCCP) and surface albedo from the Earth Radiation Budget Experiment (ERBE).

The data on precipitation is supplied by two sources: Midwest Climate Center (MCC) [*Kunkel et al.*, 1990] and EarthInfo Inc. The MCC data set consists of daily precipitation at 129 stations within the state of Illinois. The EarthInfo data set consists of hourly precipitation at 52 stations within the same area. The data on soil moisture was collected by Illinois State Water Survey (ISWS) [*Hollinger and Isard*, 1994]. Starting in 1981 ISWS collected measurements on the soil moisture content at eight grass-covered sites around Illinois using neutron probe technology. Seven additional sites were added in 1982, two more in 1986, and by 1992 the total had increased to 19. Fifteen of these 19 sites cover the period 1983 to 1994, and will be used in this study. Weekly (March–October) and biweekly (November–February) measurements of soil moisture content were taken at 11 different soil layers with a resolution of about 20 cm down to 2 m below the surface. The data on groundwater level consists of monthly groundwater level at 18 wells scattered throughout the state of Illinois for monitoring of the

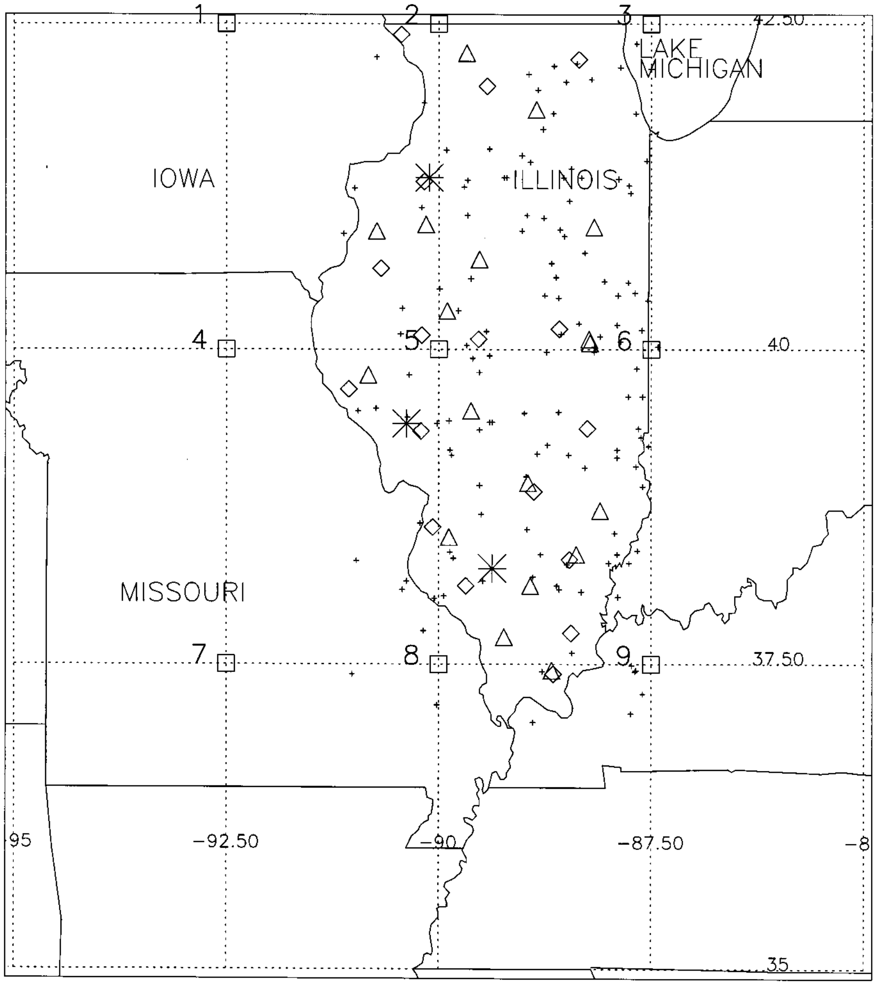

Figure 2. Location of the measurement sites of hydrological variables: precipitation (pluses), soil moisture (triangles), groundwater level (diamonds), and streamflow (stars). The nine squares at boundaries are the 2.5° × 2.5° grid points of NCEP/NCAR specific humidity and wind velocity data. The study region is encompassed by these nine grid points with an area of about 240,000 km^2.

unconfined silt loam aquifers. These aquifers are relatively shallow and the average depth to the water table ranges between 1 and 10 m below the surface. The hydrogeology of Illinois is described briefly in Appendix A.

The data on river flow have been collected by the U.S. Geological Survey and consist of daily discharge measurements at three stations: Illinois river at Valley City, which drains about 69,000 km^2; Rock River near Joslin, which drains about 25,000 km^2; and Kaskaskia River near Venedy Station, which drains about 11,000 km^2. The total drainage area of these three rivers represents roughly about two thirds of the area of the state of Illinois.

The time series describing the spatial averages of monthly atmospheric moisture convergence, precipitation, soil moisture content, groundwater level, and river flow for the common period (1983–1994) are presented in Figure 3. These variables were estimated by the arithmetic averages of each of the following observed variables from the three stations: precipitation, soil moisture content in the top 2 m, groundwater level, and river flow per unit area. It has been assumed that these averages are representative of the state of Illinois. This assumption is likely to be true for precipitation and river flow, but it is less accurate for soil moisture content and groundwater level. For a discussion on the accuracy of the estimates of atmospheric moisture convergence we refer to work by *Yeh et al.* [1998]. In the following section we present the climatology of the average regional hydrological cycle in Illinois that is based on these observations.

3. Climatology of the Regional Hydrological Cycle

In describing the climatology of the regional hydrological cycle in Illinois, we start by describing the annual water balance that is presented in Table 1. Rainfall over Illinois is 975 mm/yr; 70% of that water is returned to the atmosphere through evaporation, with runoff accounting for the remaining 30%, which is 314 mm/yr. The convergence of atmospheric moisture towards this region is 294 mm/yr. The difference between convergence and runoff is about 20 mm/yr, which is comparable to the accuracy of the estimates themselves. Since for a consistent water balance of the atmosphere and of the soil these two fluxes have to be equivalent, the above comparison suggests

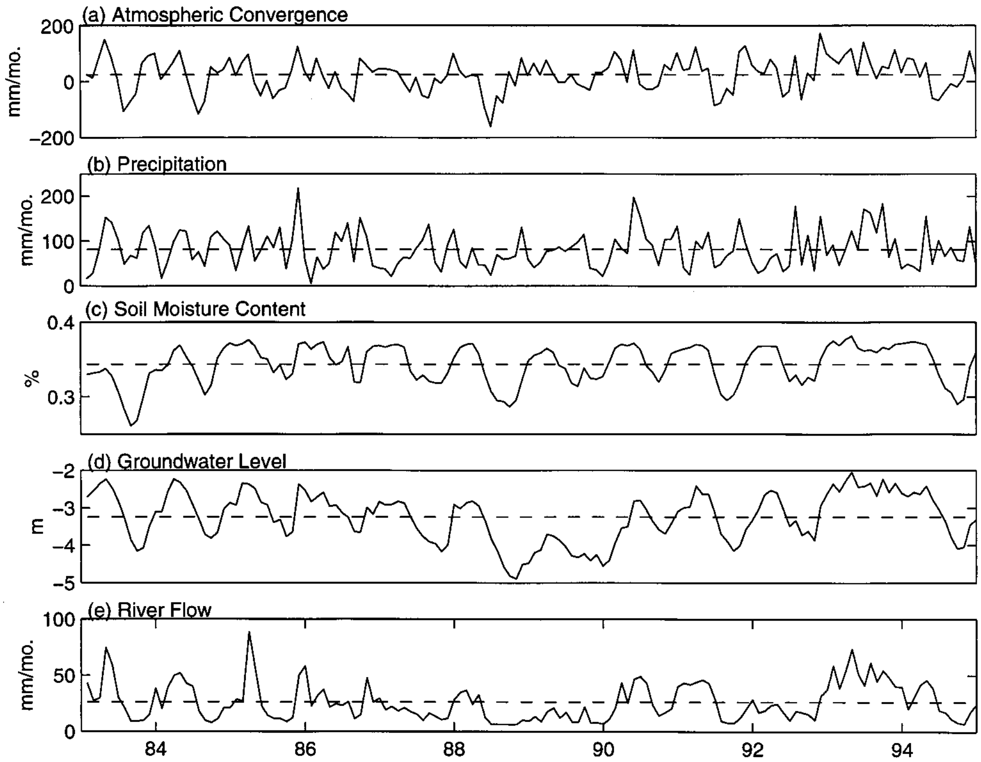

Figure 3. Twelve-year time series of monthly (a) atmospheric water vapor convergence, (b) precipitation, (c) soil moisture content, (d) groundwater level, and (e) river flow in Illinois. The dashed lines shown in the figure denote the long-term average from 1983 to 1994.

that any additional lateral land surface fluxes or leakages are indeed of negligible magnitude.

The regional hydrological fluxes exhibit significant seasonal variability. Table 2 summarizes the climatology of the soil water balance components and atmospheric water balance components. Figures 4 shows the seasonal variability in precipitation, evaporation, river flow, and atmospheric moisture convergence as well as the changes in the subsurface storage of water. Evaporation has been estimated by *Yeh et al.* [1998] using the mass balance method applied to the atmospheric and soil branches of the regional hydrological cycle. The climatology of the two evaporation estimates agree reasonably well and within an error of about 10 mm/month (see Table 2). The details of this estimation procedure and the corresponding results are given by *Yeh et al.* [1998]. The two different estimates have been combined to produce the climatology of the average evaporation that is presented in Figures 4 and 5. Rainfall is uniform throughout the year, although conditions during the spring (March, April, and May), summer (June, July, and August), and fall (September, October, and November) seasons are relatively wet in comparison with the relatively dry conditions during the winter season (December, January, February). Evaporation has a strong seasonal cycle with high evaporation rates (>120 mm/month) during the summer season (June–August) and practically no evaporation during the winter season. During the summer, the evaporation rates in this region are similar in magnitude to the high evaporation rates that have been observed by *Shuttleworth* [1988] for the Amazon rainforest. During the summer months the evaporation rate exceeds the precipitation rate, atmospheric moisture diverges away from this region, and the subsurface storage of water acts as a significant source of water to the atmosphere. However, these patterns are reversed during the remaining months of the year: precipitation exceeds evaporation and a significant amount of atmospheric moisture converges towards the region, which helps to replenish the subsurface storage of water before the onset of dry conditions in the following summer.

Table 1. Average Annual Water Balance in Illinois From 1983 to 1994

	Average
P, mm/yr	975
E, mm/yr	671
R, mm/yr	314
C, mm/yr	294
s, %	34.4
H, m	−3.24

P, precipitation; E, evaporation; R, streamflow; C, atmospheric water vapor convergence; s, soil moisture content; and H, groundwater level.

The strong seasonal cycles of incoming solar radiation, evaporation, soil moisture, groundwater level, and river flow are related to each other through the hydrological cycle (Figure 5). The strong seasonal cycle of incoming solar radiation leaves its signature on the soil moisture content through evaporation. The difference between infiltration and evaporation from the soil represents the natural replenishment rate of the soil water reservoir. Since precipitation shows no clear seasonal pattern, the seasonal cycle of evaporation leaves a clear signature on the seasonal cycle of soil moisture content as seen in Figure 5. The recharge from the soil water reservoir to the groundwater

Table 2. Average Monthly Soil Water Balance Components and Atmospheric Water Balance Components in Illinois From 1983 to 1994

	P, mm	E_{soil}, mm	R, mm	s, %	H, m	$nD(ds/dt)$, mm	$S_y(dH/dt)$, mm	$E_{atmo.}$, mm	C, mm
Jan.	39.2	1.5	27.7	35.9	−3.10	7.2	2.9	0.4	41.4
Feb.	50.2	−2.5	28.7	36.3	−2.89	7.5	16.5	−0.4	50.2
March	77.1	12.3	39.5	36.7	−2.69	8.9	16.4	12.6	66.6
April	90.0	59.0	41.2	36.4	−2.73	−6.9	−3.3	51.7	45.8
May	94.8	108.7	32.3	34.8	−2.91	−31.9	−14.3	96.9	2.8
June	89.3	112.0	24.6	33.5	−3.17	−26.2	−21.0	122.5	−22.4
July	97.3	117.5	20.9	32.5	−3.43	−20.6	−20.5	126.9	−32.0
Aug.	83.5	114.6	14.8	31.4	−3.73	−21.9	−23.9	120.4	−42.4
Sept.	89.0	80.5	15.2	31.5	−3.83	1.8	−8.6	74.1	7.9
Oct.	83.0	42.5	16.6	32.6	−3.83	23.8	0.1	50.6	31.1
Nov.	112.5	9.9	22.7	34.8	−3.38	43.5	36.4	21.0	82.9
Dec.	69.0	3.8	30.0	35.8	−3.19	19.8	15.3	5.5	62.1
Average	81.2	55.0	26.2	34.4	−3.24	0.42	−0.33	56.9	24.5
Total	974.8	659.8	314.1	···	···	4.98	−4.01	682.2	293.7

P, precipitation; E_{soil}, evaporation estimated from soil water balance; R, river flow; s, soil moisture content; H, groundwater level; $nD(ds/dt)$, change in unsaturated (soil) storage; $S_y(dH/dt)$, change in saturated (aquifer) storage; $E_{atmo.}$, evaporation estimated from atmospheric water balance; and C, convergence of atmospheric moisture.

reservoir is a slow process regulated by the variability in the soil moisture content. Hence, as seen in Figure 5, the seasonal cycle for the groundwater level lags that of soil moisture content by about one month, which is consistent with the conclusion of *Changnon et al.* [1988]. The seasonal cycle of river flow follows closely that of groundwater level as seen in Figure 5. Examination of the seasonal cycle of snow depth (not shown; see Figure 3 of *Yeh et al.* [1998]) suggests that snow melting in spring has a relatively small magnitude and is likely to have a minor impact on river flow. As a result, the peak of river flow during the spring season is attributable to the high groundwater levels during that season.

In summary, the strong seasonal cycle of solar radiation forces a similar cycle in evaporation, which then propagates through the soil hydrology to dictate the observed patterns in the seasonal variability of soil moisture content and ground-

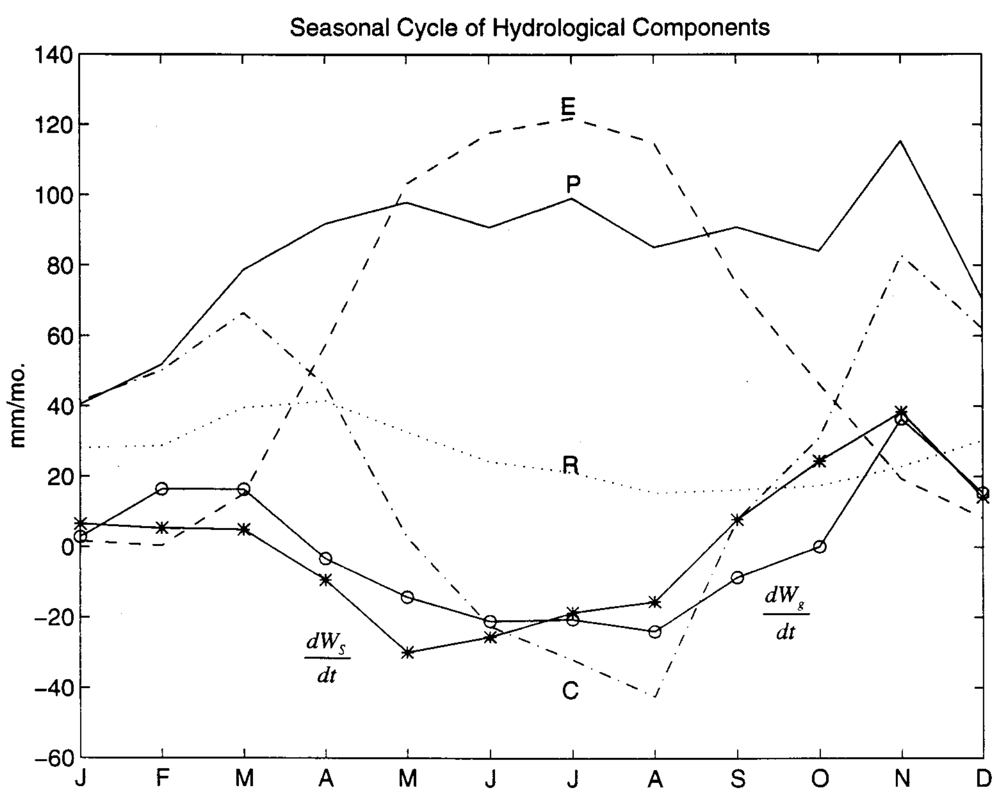

Figure 4. Seasonal cycle of hydrological components in Illinois: P, precipitation; E, evaporation; R, river flow; C, convergence of atmospheric moisture; dW_S/dt, change in unsaturated (soil) storage; and dW_g/dt, change in saturated (aquifer) storage.

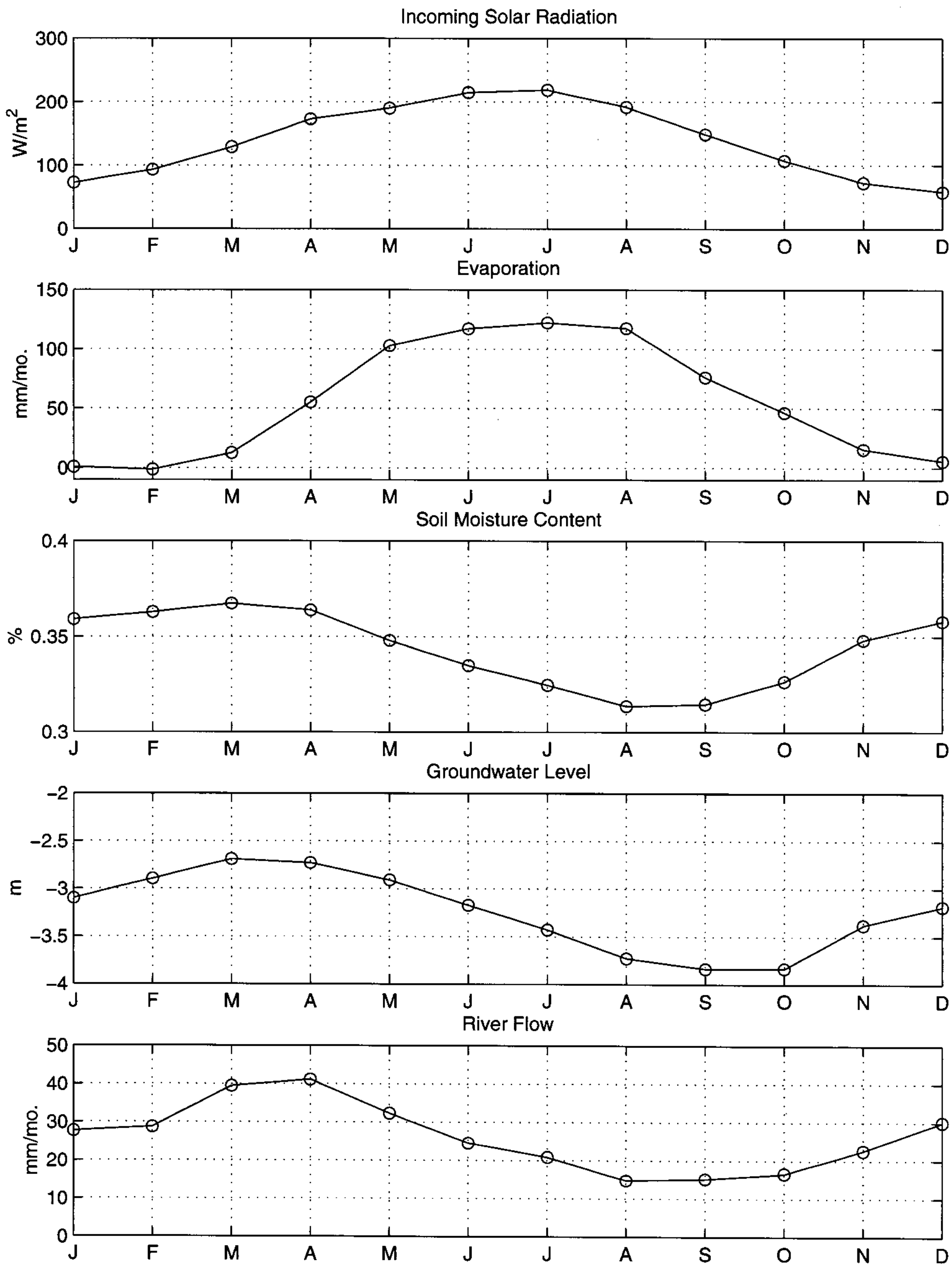

Figure 5. Seasonal cycle of incoming solar radiation, evaporation estimate, soil moisture content, groundwater level, and river flow.

water level. The seasonal cycle of river flow follows closely that of groundwater level, which suggests a significant contribution by the groundwater runoff to observed river flow in Illinois.

4. Anomalies in the Regional Hydrological Cycle: Floods and Droughts

The previous section describes the climatology of the spatially averaged regional hydrological cycle of Illinois. In this section we consider standardized anomalies of these observed variables from their corresponding monthly averages. For each variable we estimate the average and standard deviation corresponding to each of the 12 months of the year. The differences between any observed variable and the corresponding monthly average for that variable and for that month of the year were first computed. Then, to compute standardized anomalies, these differences were normalized by the corresponding standard deviations. Figure 6 presents times series of the resulting anomalies in the following variables: convergence of atmospheric moisture, precipitation, soil moisture content, groundwater level, and river flow. This figure describes how the anomalies in the large-scale atmospheric circulation propagate through the hydrological cycle to determine the magnitude of precipitation, which then regulate the variability in soil moisture conditions and recharge to aquifers, resulting in a clear

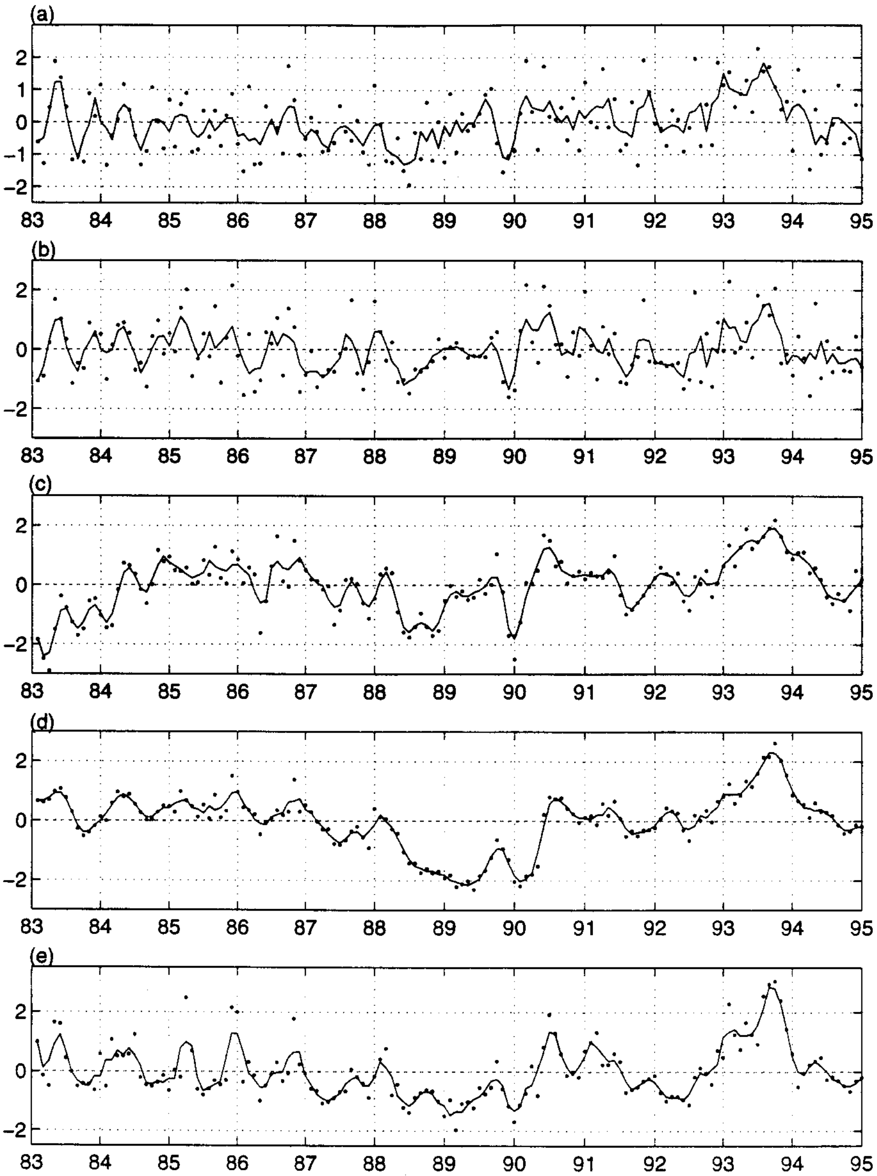

Figure 6. Time series of the normalized anomalies of monthly (dots) (a) atmospheric moisture convergence, (b) precipitation, (c) soil moisture content, (d) groundwater level, and (e) river flow from 1983 to 1994 in Illinois. The continuous line is a 12-month moving average.

signature of the circulation anomalies on the levels of groundwater aquifer and river flow. While precipitation plays a minor role in shaping the natural variability in the regional hydrological cycle at the seasonal timescale, variability in precipitation is the major factor in shaping the natural variability in the regional hydrological cycle at the interannual timescale.

The persistence characteristics of the observed hydrological anomalies have been analyzed (see Figure 7). The correlation timescales of atmospheric moisture convergence and precipitation are shorter than 1 month (i.e., within one lag), whereas the correlation timescales of soil moisture, groundwater level, and river flow are about 3, 12, and 3 months, respectively. As the circulation anomalies propagate through the system, the high-frequency contributions to the hydrological signal are smoothed out and their contributions to the overall variance are reduced (Figure 7). This is indeed true for the propagation of the signal from the atmospheric circulation to precipitation, to the soil moisture, and then to the groundwater levels. The soil water reservoir and groundwater reservoirs act as two adjacent low-pass filters of the atmospheric forcing as it propagates through the hydrological cycle. However, the fluctuations in river flow reflect the variability in precipitation (through the surface runoff production mechanisms) as well as the groundwater level (through groundwater runoff). As a result, fluctuations in river flow are richer in the high-frequency components compared to groundwater level fluctuations.

Several small-scale floods and droughts with anomaly magnitudes of about 1 standard deviation and duration of a few months characterize the first 5 years of this analysis (1983–1987). Although the magnitude of any of these events is rather small, most hydrological anomalies leave a clear signature on all the hydrologic variables considered (as seen in Figure 6).

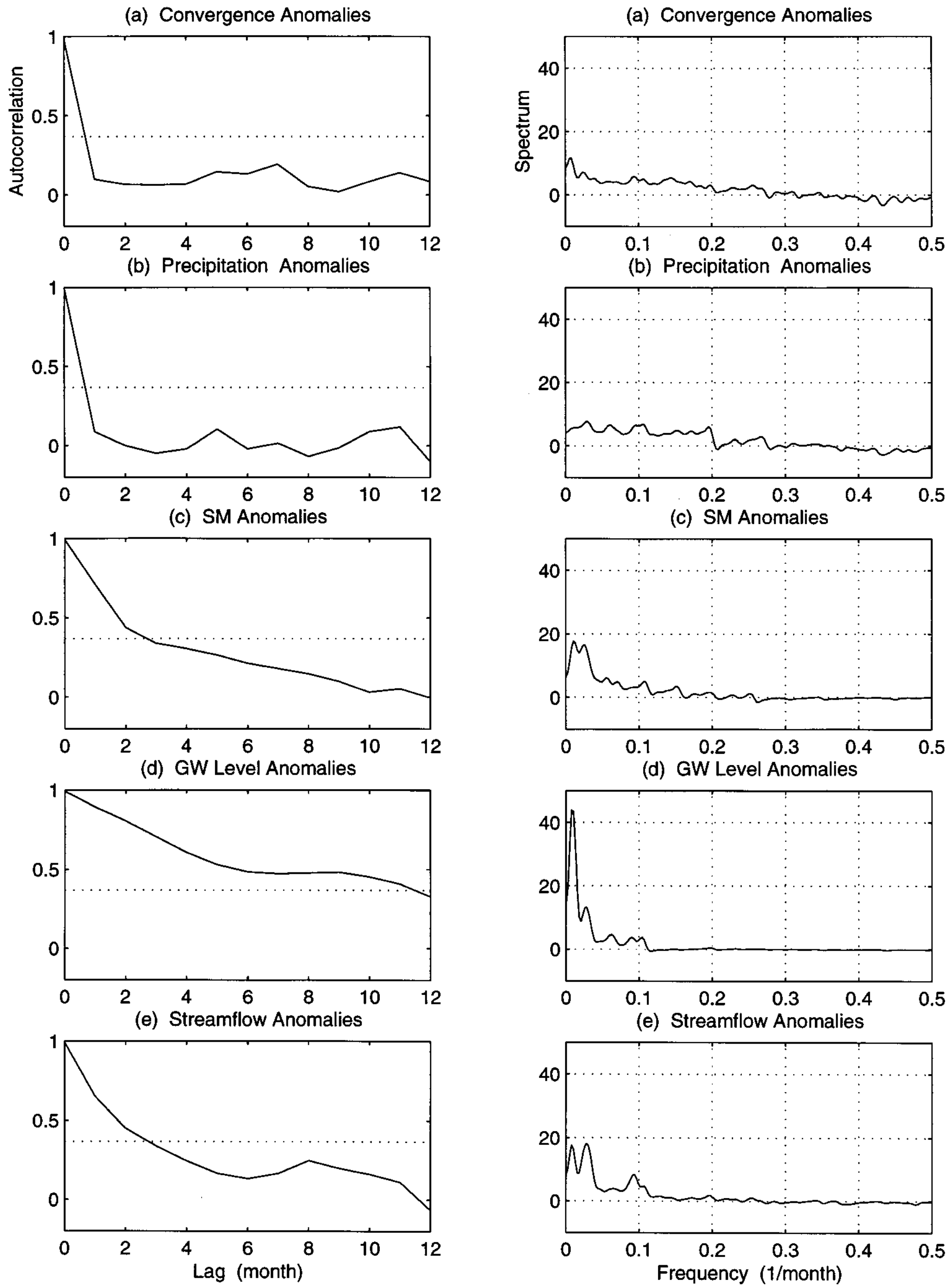

Figure 7. The autocorrelation functions and the spectra for the standardized anomalies of atmospheric moisture convergence, precipitation, soil moisture content, groundwater level, and river flow. The dotted lines denote the e-folding correlation timescale.

Moreover, the degree of persistence in these anomalies increases as they propagate through the soil hydrology, from the initial forcing by the atmospheric circulation to the eventual response by the groundwater aquifer.

The following 7 years (1988–1994) were marked by the occurrence of two extreme summer anomalies: the drought of 1988 and the flood of 1993. Both events were associated with hydrological anomalies that had magnitudes of about 2 standard deviations and extended for about a year. The response of the hydrological cycle to these two events will be examined in some detail as examples for significant summer floods and droughts. The drought of 1988 was triggered by anomalous divergence in the atmospheric circulation that started in February 1988 and continued for most of the spring and summer seasons of 1988. A similar anomaly in precipitation was observed starting early spring and extending through the spring and summer seasons. However, the corresponding soil moisture anomaly persisted longer and dry conditions were observed throughout the spring, summer, and fall seasons of 1988. The response of the groundwater level was even more dramatic, with relatively low levels persisting until the end of the fall season of the following year. Another drought episode that occurred in the fall of 1989 triggered a further decrease in the aquifer level, before full recovery during 1990. The observed anomalies in precipitation and groundwater level left clear signatures on observed river flow.

The 1993 flood was triggered by anomalous atmospheric convergence that started during the fall of 1992 and persisted for about a year, resulting in a precipitation anomaly with a similar persistence pattern. However, the corresponding soil moisture anomaly persisted throughout the spring, summer, fall, and winter seasons and well into the following year and up to the early summer of 1994. The level of water in the groundwater aquifer experienced a persistent rising trend starting in the fall of 1992 and continuing for about a year until the fall of 1993, when it leveled off, declined sharply, and eventually returned to normal levels during the spring of 1994. The corresponding pattern of river flow anomaly was very similar to that of the groundwater level anomaly suggesting a strong association between the two variables during this wet episode.

The patterns of hydrological anomalies in soil moisture content for the 1988 drought and 1993 flood will be compared. The 1993 flood left a significantly more persistent signature on the soil moisture content when compared to the signature of the 1988 drought. While the soil moisture content returned to its normal levels by the end of 1988, a similar recovery did not occur until the early summer of 1994. Although some difference in the recovery patterns would be expected as a result of the different characteristics of the two precipitation anomalies, the large differences between the observed patterns invite additional explanations. The climatology of the regional hydrological cycle that has been presented in section 3 indicates that the fall and winter seasons are the wet seasons for soil hydrology, when precipitation exceeds evaporation by a large magnitude. This surplus helps in replenishing the soil water reservoir, from the normally dry summer conditions to the normally wet winter conditions, and then in recharging the excess water to the groundwater reservoir. Following the summer drought of 1988, the soil moisture recovered to normal levels quickly; however, the aquifer water level continued to decline throughout the following fall and winter seasons. This contrast suggests that the normal rainfall levels observed during the fall and winter seasons of 1988, were consumed in replenishing the anomalous dry summer conditions back to the normal winter soil moisture conditions, at the expense of the normal rate of recharge to the aquifer water levels. As a result the aquifer water levels continued to decline as observed. In comparison, under the usual conditions of low evaporation during the fall and winter of 1993, the slow recharge from the soil water reservoir to the aquifers was not efficient enough to dissipate the wet soil moisture anomaly. As a result, the wet soil moisture anomaly persisted until the onset of the following summer season.

The response of the aquifer water level to the 1988 drought and 1993 flood was exactly opposite that of soil water reservoir. A more persistent dry anomaly was observed in 1988 compared to a less persistent wet anomaly in 1993. The observations on river flow show a wet anomaly of extreme magnitude and short duration that took place in the late summer and early fall of 1993. This event was followed by a sharp decline in aquifer water level, which suggests that groundwater runoff acts as an efficient dissipation mechanism of wet anomalies in aquifer water levels. The association between anomalies in river flow and aquifer water level is less evident during the long recession of aquifer level in 1988 and 1989. There is no apparent negative feedback that could act as a dissipation mechanism for the drought conditions.

5. Statistical Analysis of the Hydrological Anomalies Using the Crossing Theory

In the previous section we presented observations on the persistence patterns of hydrological anomalies in Illinois and suggested that aquifers respond differently to floods and droughts. More rigorous analyses of the observations on precipitation and aquifer water level have been performed to provide additional evidence for the asymmetric response of aquifers to floods and droughts. The crossing theory [*Bras and Rodriguez-Iturbe*, 1985; *Nordin and Rosbjerg*, 1970] provides a powerful set of statistical analysis tools that are ideally suited for analyzing the characteristics of floods and droughts. In the following we present a short review of the main concepts of the crossing theory followed by an application of that theory to study the characteristics of floods and droughts in Illinois.

5.1. The Crossing Theory

The crossing theory was first developed by *Rice* [1945] for analysis of the statistical properties of random noise and was extended by *Cramer and Leadbetter* [1967]. The previous applications of the crossing theory in hydrology have been summarized by *Bras and Rodriquez-Iturbe* [1985, pp. 240–261]. The theory deals with the properties of excursions of random processes above (i.e., floods) or below (i.e., droughts) a certain threshold value of the process [*Bras and Rodriguez-Iturbe*, 1985, pp. 240–261; *Nordin and Rosbjerg*, 1970]. It provides a definitive measure of persistence in hydrological time series. Figure 8, taken from *Bras and Rodriguez-Iturbe* [1985], shows a discrete time series, $Y(t)$, normalized to have a zero mean and a unit variance. The quantities of interest in the figure are (1) the time between successive upcrossings or downcrossings (B_u^+ or B_u^-), which measures the frequency of occurrence of floods or droughts; (2) the interval between zero crossings (l_0), which measures the persistence of floods (droughts); (3) the duration of excursions above or below a certain threshold value (l_u^+ or l_u^-), which measures the duration of the corresponding hydrological anomaly (floods or droughts); and (4) the area of an excursion above or below the threshold level (A_u^+ or A_u^-), which measures the cumulative magnitude of a hydrological anomaly above a threshold value, u. Given a hydrologic time series with sufficient length, all these crossing statistics, B_u^+, B_u^-, l_0, l_u^+, l_u^-, A_u^+, and A_u^-, can easily be estimated.

The analytical formulae for describing these crossing statistics have been developed for Gaussian processes. For a non-Gaussian process (with skewness coefficient larger than about 0.2), significant differences from the theory would be anticipated depending on the degree of the deviation from normality assumption [*Nordin and Rosbjerg*, 1970]. In this paper we compare the characteristics of observed floods and droughts to the corresponding theoretical values for a Gaussian stochastic process that shares with the observed anomalies series the same mean, standard deviation, and autocorrelation coefficient at lag 1. The Gaussian process is, by definition, symmetric in all of its statistical characteristics. Thus comparing observations with a compatible Gaussian process reveals the asymmetric aspects in the observed patterns of floods and droughts.

The expected value of A_u^+ can be estimated by dividing the area above an excursion level u by the number of excursions

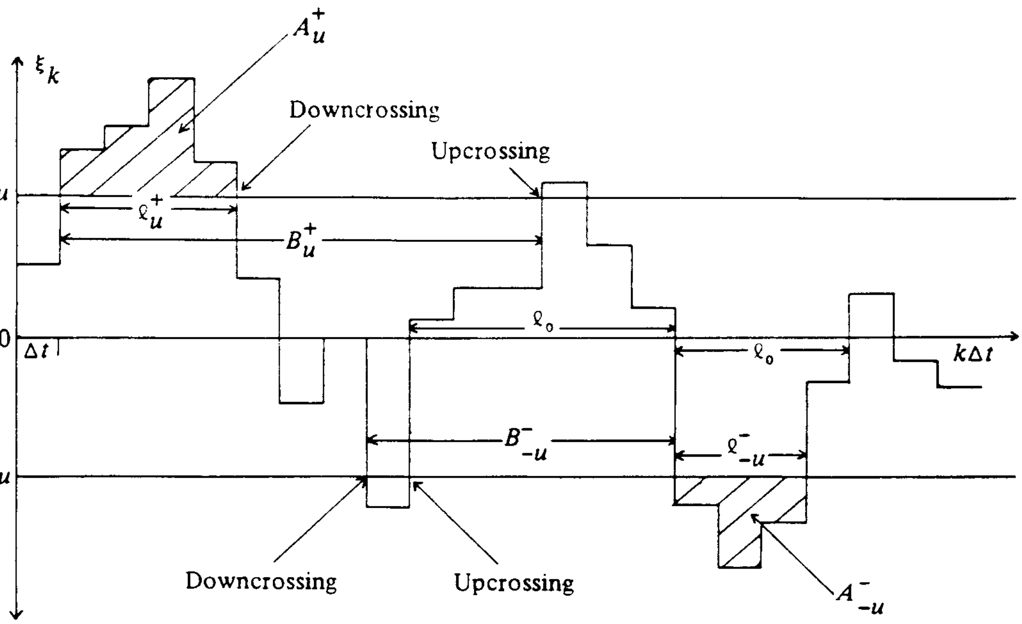

Figure 8. Discrete time series, normalized to have zero mean and unit variance. The figure shows all the relevant variables that are needed to describe a time series using crossing theory (see text for the definitions of all the variables). (From *Bras and Rodriguez-Iturbe* [1985]. Copyright 1985 by Addison-Wesley; reprinted with permission.)

above u. For a continuous Gaussian process this quantity can be derived as [*Bras and Rodriguez-Iturbe*, 1985, p. 245]

$$E[A_u^+] = 2E[Z_1(T)]E[l_0]e^{u^2/2\sigma^2} \tag{1}$$

where

$$E[l_0] = \pi\left[\frac{\rho(0)}{2\rho(1) - 2}\right]^{1/2}$$

$$E[Z_1(T)] = \frac{1}{\sigma\sqrt{2\pi}}\int_u^\infty (x-u)e^{-x^2/2\sigma^2}\,dx \tag{2}$$

Slight modification is needed to apply (1) and (2) to a discrete time series, such as annual streamflow records. Since Gaussian processes are symmetrical, the theoretical expression for the expected value of the area above a threshold u is exactly the same as the that for the expected value of the area below a threshold $-u$.

A useful absolute measure of persistence is the ratio $E[l_u^+]/E[l_0]$, which measures the average duration of floods or droughts in terms of the average time between zero crossings. For a discrete process with a finite lag 1 autocorrelation this ratio can be derived theoretically as follows [*Nordin and Rosbjerg*, 1970]:

$$\frac{E[l_u^+]}{E[l_0]} = \frac{\left[\frac{1}{2} - \left(\frac{1}{\pi}\right)\arcsin\phi(1)\right]pr\{Y > u\}}{pr\{Y > u\} - pr\{Y_1 > u,\ Y_2 > u\}} \tag{3}$$

where the $\phi(1)$ is the value of autocorrelation of Y at lag 1. The probability distribution function (PDF) of Y, and the joint PDF between two arbitrary Y values, Y_1 and Y_2, are given by

$$pr\{Y > u\} = \frac{1}{\sqrt{2\pi}}\int_u^\infty e^{-(1/2)x^2}\,dx \tag{4}$$

$$pr\{Y_1 > u,\ Y_2 > u\} = \frac{1}{2\pi[1-\phi(1)^2]^{1/2}} \cdot \int_u^\infty\int_u^\infty \exp\left\{-\frac{u_1^2 - 2u_1u_2 + u_2^2}{2[1-\phi(1)^2]}\right\} du_1\,du_2 \tag{5}$$

Since Gaussian processes are symmetrical, the theoretical expression for the expected value of the duration of excursions above a threshold u is exactly the same as the that for the expected value of the duration of excursions below a threshold $-u$. The variable defined by $E[l_u^+]/E[l_0]$ in (3) provides a measure of the average duration of a hydrological anomaly, whereas $E[A_u^+]$ provides a measure of the average magnitude of a hydrological anomaly above (or below) a certain threshold level.

5.2. Application of the Crossing Theory to the Study of Floods and Droughts in Illinois

In this section we use expanded data sets on precipitation and aquifer water level covering the common period of 1970–1996 (see Figure 9). The state average variables are transformed into standardized anomalies by subtracting the monthly mean and then dividing by the corresponding standard deviation for that month. Floods (droughts) are characterized by the excursions of the anomalies above (below) a certain threshold. For floods we use positive thresholds; for droughts we use negative thresholds.

Figure 10 shows estimates of the average size of floods and droughts corresponding to different threshold levels. For precipitation, observed floods are consistently larger than the corresponding Gaussian floods, while observed droughts are smaller than the corresponding Gaussian droughts. This difference is interesting and is in general consistent with the

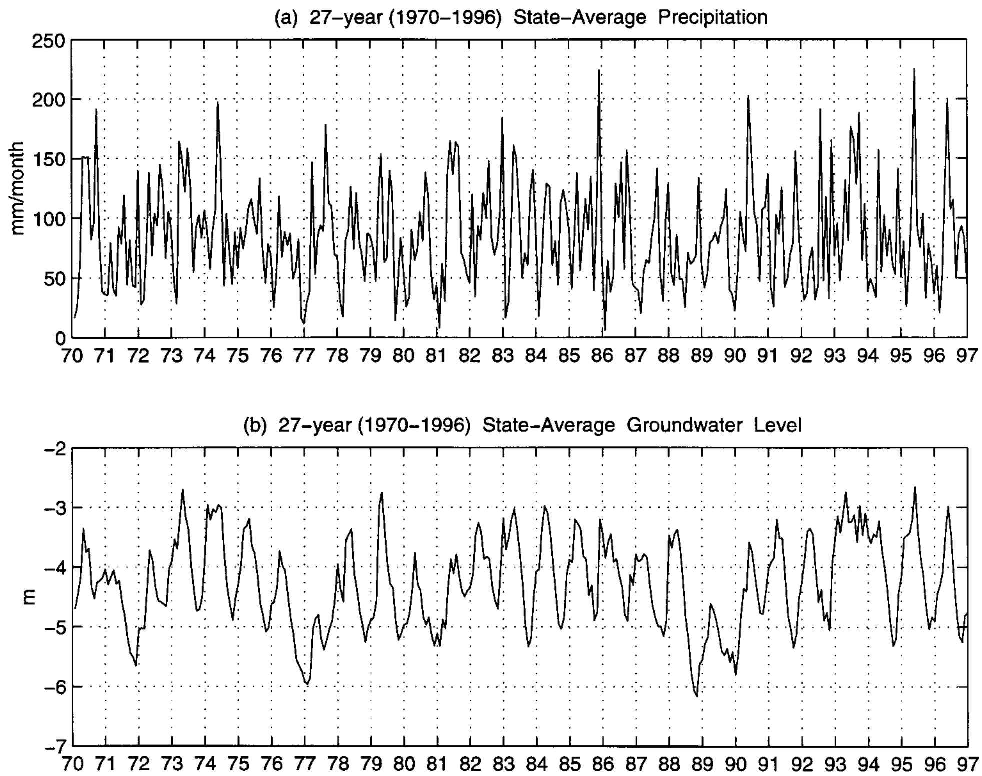

Figure 9. Time series of (a) precipitation and (b) groundwater level (1970–1996) from Illinois.

positively skewed distribution of precipitation anomalies (not shown here). Identification of the physical mechanisms that are responsible for these statistical characteristics is an open subject for future research. For the aquifer water level we observe a pattern of asymmetry that is the reverse of the pattern in precipitation anomalies: observed droughts are consistently larger than the corresponding Gaussian droughts, while observed floods are consistently smaller than the corresponding Gaussian floods.

A similar analysis was performed on another important characteristic of the observed floods and droughts: duration. We compared observed duration to the corresponding expected duration for Gaussian floods and droughts (see Figure 11). Again we see a reversal in the pattern of the observed asymmetry when comparing precipitation and aquifer water level. Droughts (floods) are more (less) persistent in observed aquifer water level than in the corresponding Gaussian process. However, for the same time period, droughts (floods) are less (more) persistent in observed precipitation than in the corresponding Gaussian process. This observed tendency for an increase (decrease) in the persistence and magnitudes of droughts (floods) as they propagate from the atmosphere to the aquifer is an important characteristic of the regional hydrologic cycle.

6. Physical Basis of the Asymmetric Response of Aquifers to Floods and Droughts

In the previous sections we presented empirical evidence for the asymmetric response of aquifers to floods and droughts in Illinois. In this section we analyze the physical mechanisms that are responsible for these empirical observations. We propose that the nonlinear form of the groundwater rating curve is the key factor responsible for the different patterns of persistence that have been observed during floods and droughts. This nonlinearity is attributed mainly to the interactions between the aquifer water level and the density of the channels network.

6.1. The Link Between the Patterns of Persistence During Floods and Droughts and the Nonlinear Form of the Groundwater Rating Curve

Here we perform a simple analysis of groundwater level fluctuations to support the hypothesis that the observed asymmetric response of aquifers is a natural result that follows from the nonlinear form of the groundwater rating.

First, we need to characterize the groundwater rating curve for Illinois. Examination of the observations on precipitation, aquifer water level, and river flow suggests that monthly river flow is significantly more correlated to aquifer levels than to precipitation. While aquifer water level explains more than 60% of the variance of monthly river flow, precipitation fails to explain more than 10% of the same variance (see Figure 12). Since precipitation is the main forcing of surface runoff, R_S, and aquifer level, H, is the main forcing of groundwater runoff (base flow), R_g, we conclude that $R_g \gg R_S$. On the basis of linear regression analysis performed on precipitation, groundwater level, and river flow, we estimate that R_g contributes about 75% of river flow, and for this reason observed river flow is a good surrogate for groundwater runoff. Hence we assume that Figure 12b describes reasonably well the general features of the relationship between R_g and H. Similar observations on the nonlinear nature of the groundwater rating curve have been reported by *Lizarraga* [1978] and *Senn* [1980] for New

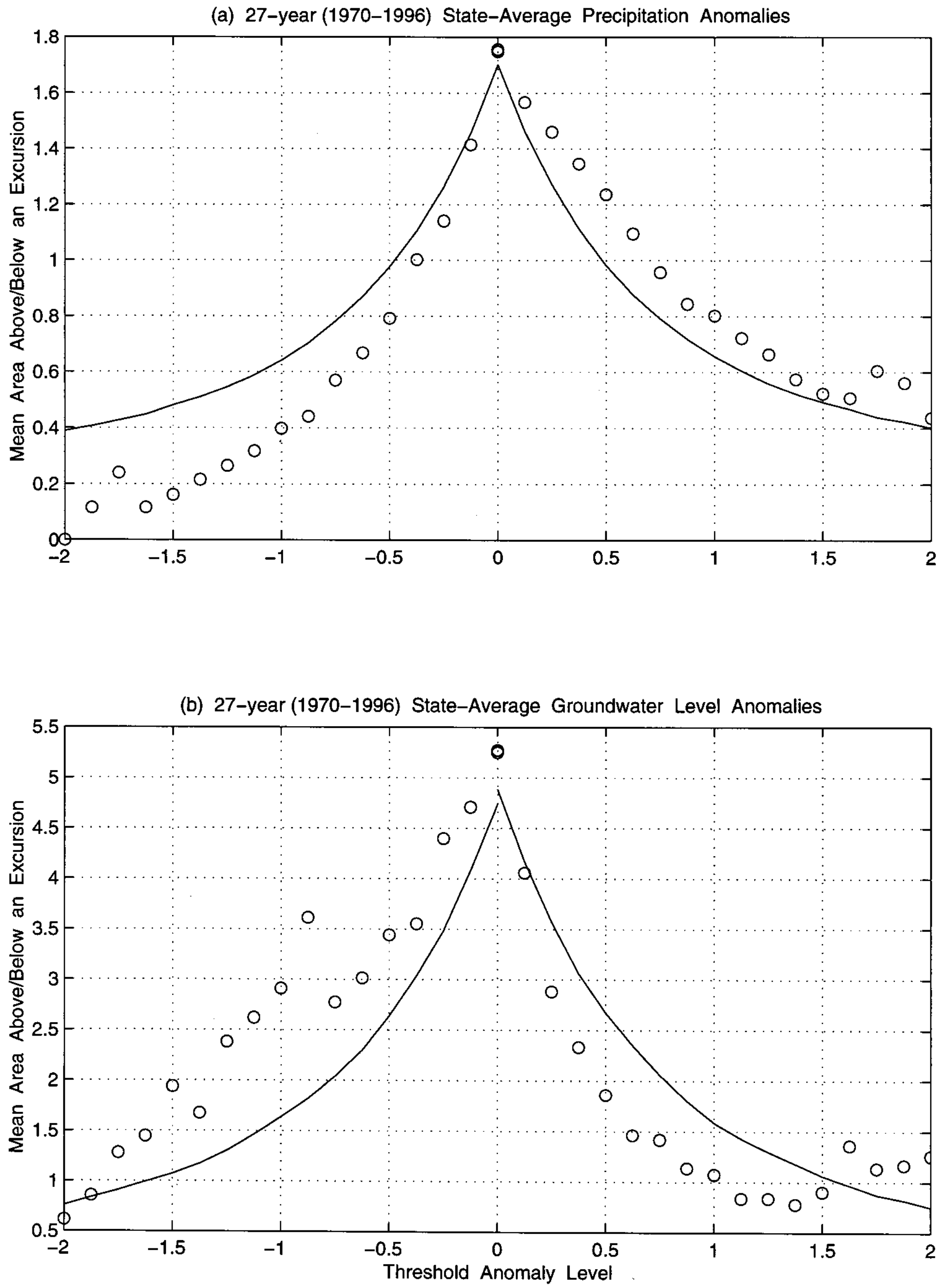

Figure 10. Mean area above (below) an excursion corresponding to positive (negative) threshold levels for (a) precipitation and (b) groundwater level. Circles are estimates from the observations of Figure 9. Continuous lines describe the theoretical relations for a symmetrical Gaussian stochastic process that has the same mean, standard deviation, and autocorrelation coefficient at lag 1.

Mexico, by *Rasmussen and Andreasen* [1959] for Maryland, and by *Schicht and Walton* [1961] for Illinois.

In Figure 12b, starting with the median of the observed aquifer water levels, changes in H toward higher water levels are associated with a relatively large increase in observed river flow per unit rise in aquifer water level. However, changes in H toward lower water levels are associated with a significantly smaller sensitivity of observed river flow. Precisely for this reason, we propose that groundwater aquifers act selectively to dissipate the wet anomalies in aquifer water levels such as those experienced in 1993. Under the relatively dry conditions similar to those of 1988 and 1989 the groundwater runoff does not respond significantly to the drop in aquifer water levels (see Figure 6). The only recovery mechanism for aquifer water level is through occurrence of anomalous wet conditions, such as those experienced during the winter and spring seasons of 1990 leading to the eventual recovery from the drought conditions.

The hypothesis that groundwater aquifers act selectively to dissipate wet anomalies, resulting in the observed asymmetric

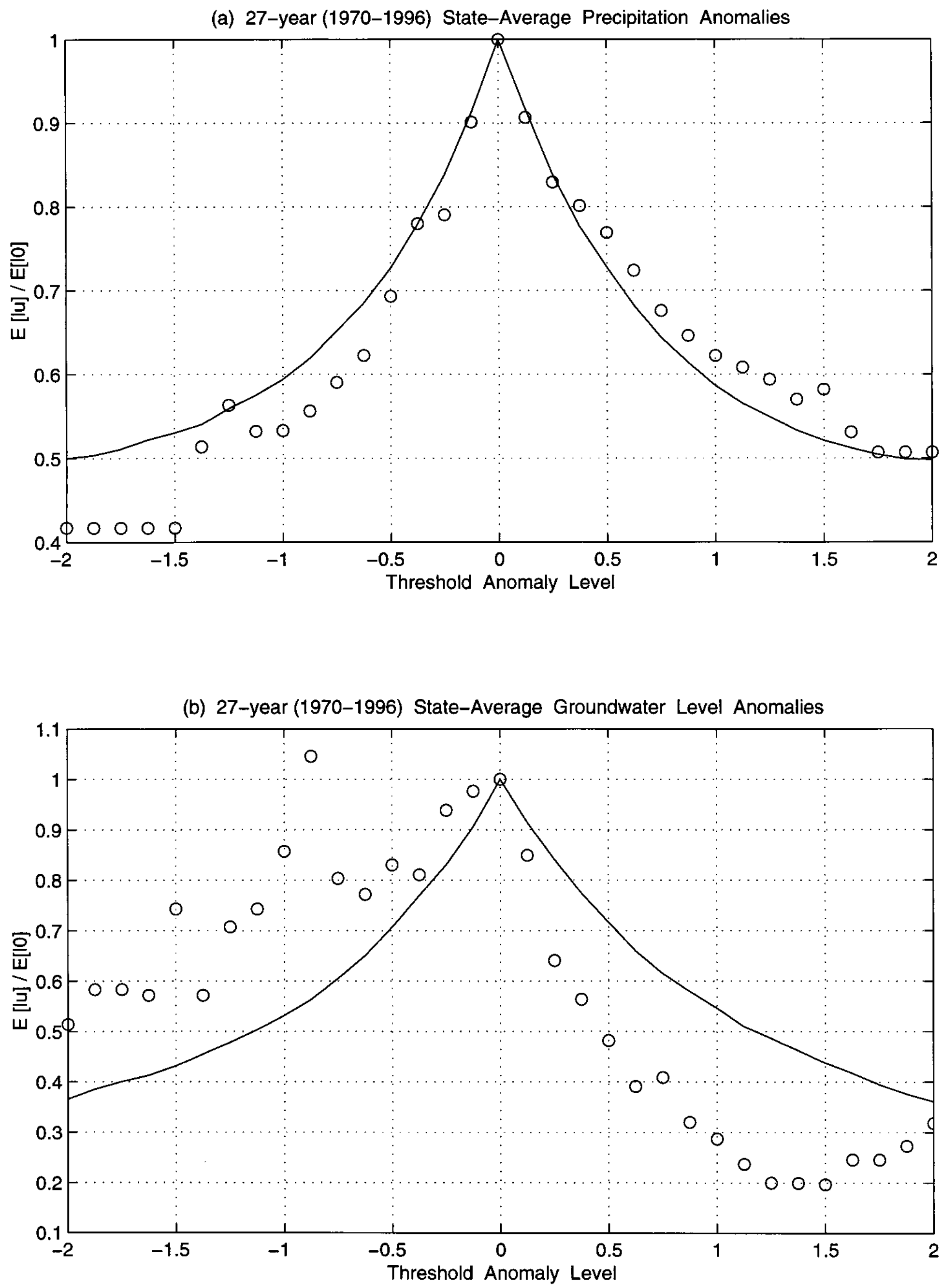

Figure 11. Normalized mean duration above (below) an excursion corresponding to positive (negative) threshold levels for (a) precipitation and (b) groundwater level. Circles are estimates from the observations of Figure 9. Continuous lines describe the theoretical relations for a symmetrical Gaussian stochastic process that has the same mean, standard deviation, and autocorrelation coefficient at lag 1.

response of the aquifer water level to floods and droughts, has been studied using a simple model of the groundwater aquifer. A lumped model of water level fluctuations in an unconfined aquifer is described by

$$S_y \frac{dH}{dt} = G - Q_g \tag{6}$$

where S_y is the specific yield, H is the state average groundwater level, G is the percolation (recharge) to the groundwater aquifer, and Q_g is the rate of groundwater runoff. In addition, we consider the following simple linear function to relate Q_g and H:

$$Q_g = Q_0 + K(H - H_0) \tag{7}$$

where H_0 is a constant reference level, Q_0 is the corresponding discharge level, and K is an outflow coefficient which is inversely proportional to the timescale of aquifer response. The observed relationship between Q_g and H is nonlinear (see Figure 12b). For this reason (7) is not suitable to describe the observed aquifer system in Illinois. However, this linear func-

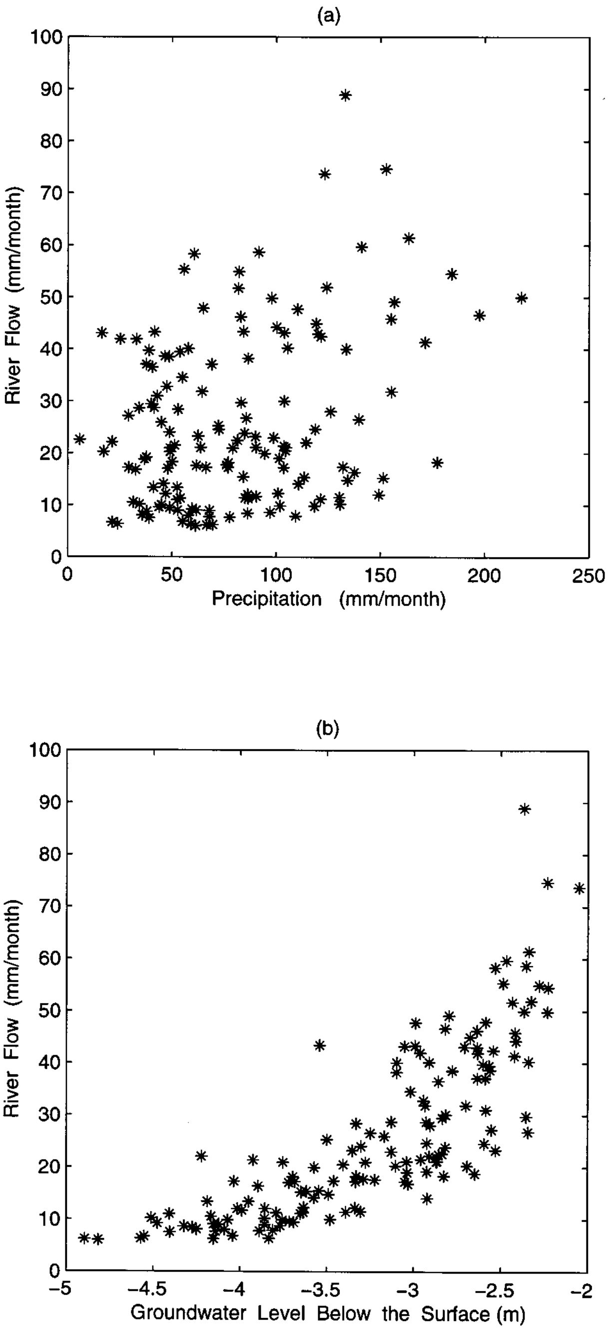

Figure 12. Plots of monthly (a) precipitation and (b) groundwater level versus monthly river flow during 1983–1994 in Illinois.

tion approximates the different segments in the observed relationship between Q_g and H such that K increases as we move from drought conditions into flood conditions. Studying the behavior of this simple system illustrates how the nonlinear form of the relationship between Q_g and H is responsible for the asymmetry in the response of the aquifer to floods and droughts. In order to estimate typical values of K, the data points in Figure 12b are divided into two groups: $H > -4$ m and $H < -4$ m. Then, two values of K were estimated by linear regression analysis: $K = 0.006$ for the dry conditions (droughts) and $K = 0.025$ for the wet conditions (floods), a factor of 4 difference between wet and dry conditions (see Figure 13).

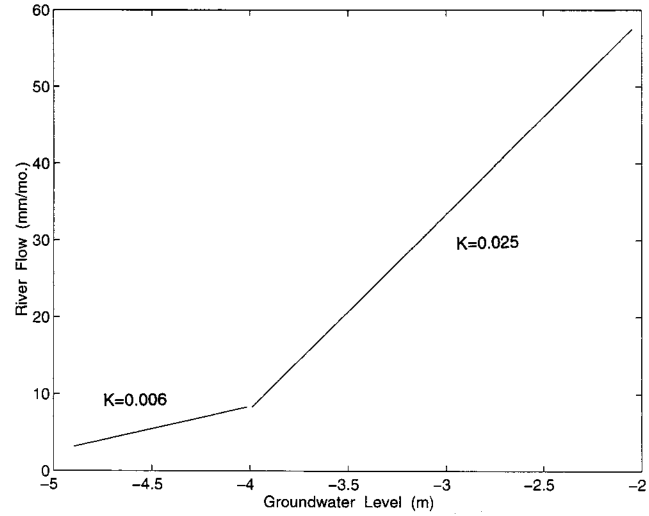

Figure 13. Linear representation of the relationship between river flow and groundwater level obtained by performing a linear regression analysis of the data of Figure 12b.

The combination of (6) and (7) results in a simple linear model for aquifer level fluctuations as follows

$$S_y \frac{dH}{dt} = G - [Q_0 + K(H - H_0)] \tag{8}$$

This equation will be analyzed using stochastic analysis techniques after assuming that natural variability in recharge can be described by a stochastic process. The same equation can be analyzed deterministically. We choose to apply statistical analysis tools in order to achieve a realistic description of the natural variability of the groundwater system. The model of (8) is analogous to a first-order Markov process except that the forcing G is not limited to be a pure random process. A similar model has been used to describe the response of phreatic aquifers to natural recharge by *Gelhar* [1993, pp. 64–70], assuming the outflow coefficient K to be a constant. Here we will consider the following two cases: K is large, corresponding to flood conditions, for which (8) describes a strongly dissipative system, and K is small, corresponding to drought conditions, for which (8) describes a weakly dissipative system.

The stochastic analysis of (8) is described in Appendix B. We assume that the forcing G is a stochastic process described by an exponential covariance function. Figure 14 shows the resulting covariance function of groundwater level $R_{hh}(\tau)$ for several values of outflow coefficient K. As K increases from 0.006 to 0.025 (see Figure 13) the correlation timescale of groundwater level decreases from about 14 months to 5 months. This is consistent with the observed difference in the persistence patterns of floods and droughts in Illinois. The dependence of the correlation timescale of groundwater level on K is shown in Figure 15 for a range of K values covering different conditions from droughts to floods. Decreasing K from $K = 0.1$ to $K = 0.01$ increases gradually the correlation timescale of groundwater level from about 1 to 9 months. A further decrease in K results in a sharp increase in correlation timescale to a few years.

Functional expressions (other than (7)) that describes the

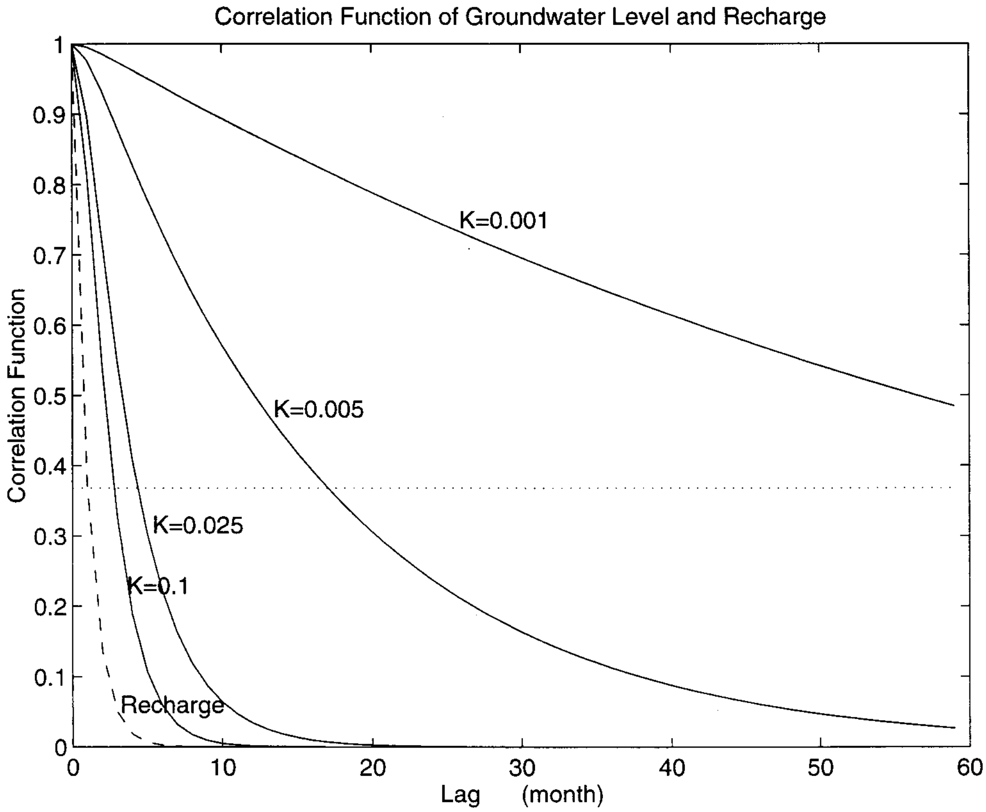

Figure 14. The autocorrelation function of groundwater level for several values of outflow constant K. The dotted line denotes the e-folding correlation timescale. (The correlation timescale of recharge is assumed to be 1 month.)

nonlinear relationship between groundwater runoff and groundwater level have been incorporated into the stochastic analysis using linearization technique (not shown here). Although quantitative differences exist, the response of the groundwater aquifer to different anomalies is qualitatively similar regardless of the function used to describe the relationship between groundwater runoff and groundwater level. Similar results have also been obtained by assuming other types of covariance function (e.g., hole-type function) for the recharge instead of the exponential function in (B4). In summary, the asymmetric response of groundwater aquifers can be explained as being due to the nonlinear form of the groundwater rating.

6.2. Physical Basis of the Nonlinear Groundwater Rating Curve

Here we would like to discuss the physical basis of the nonlinearity in the groundwater rating curve. We consider a simple unconfined rectangular aquifer placed on a horizontal impermeable layer draining into a fully penetrating stream (see Figure 16). Although this model is rather simple in comparison to the complexity of the natural system, studying the basic behavior of this model may offer some insight regarding the main physical factors in shaping the form of the groundwater rating curve. *Troch et al.* [1993] developed the following asymptotic relationship for describing the discharge into a channel of length L:

$$q_g \sim k h_r^2 D_d L \tag{9}$$

where k is the hydraulic conductivity, h_r is the average aquifer water level in this rectangular aquifer, D_d is the drainage density (total length of channels per unit area), and L is the total length of the channels. The same relationship can be expressed differently to describe the discharge per unit area,

$$Q_g \sim k h_r^2 D_d^2 \tag{10}$$

The dynamics of base flow in this model reflect the dynamics in aquifer water level fluctuations and the density of the drainage network, which is assumed to be constant in this simple model. These two factors appear explicitly in the above equation. The direct dependence of base flow on the average aquifer water level is regulated by the relative size of the fluctuations in aquifer water level to the average depth of the aquifer. In locations where this ratio is relatively large, fluctuations in aquifer water level are reflected significantly in h_r and base flow. Otherwise, these fluctuations would have a small impact on these variables and the direct dependence of base flow on aquifer water level can be approximated by a linear relationship.

However, in natural systems drainage density is a dynamic variable which varies significantly as the channels network expands and contracts in response to fluctuations in aquifer water level. Thus the following connection, rise in aquifer water level $\rightarrow$ enhancement of drainage density $\rightarrow$ increase in base flow, may provide an indirect pathway for relating aquifer water level and base flow. This pathway would tend to enhance, and under most conditions would even dominate, the relation-

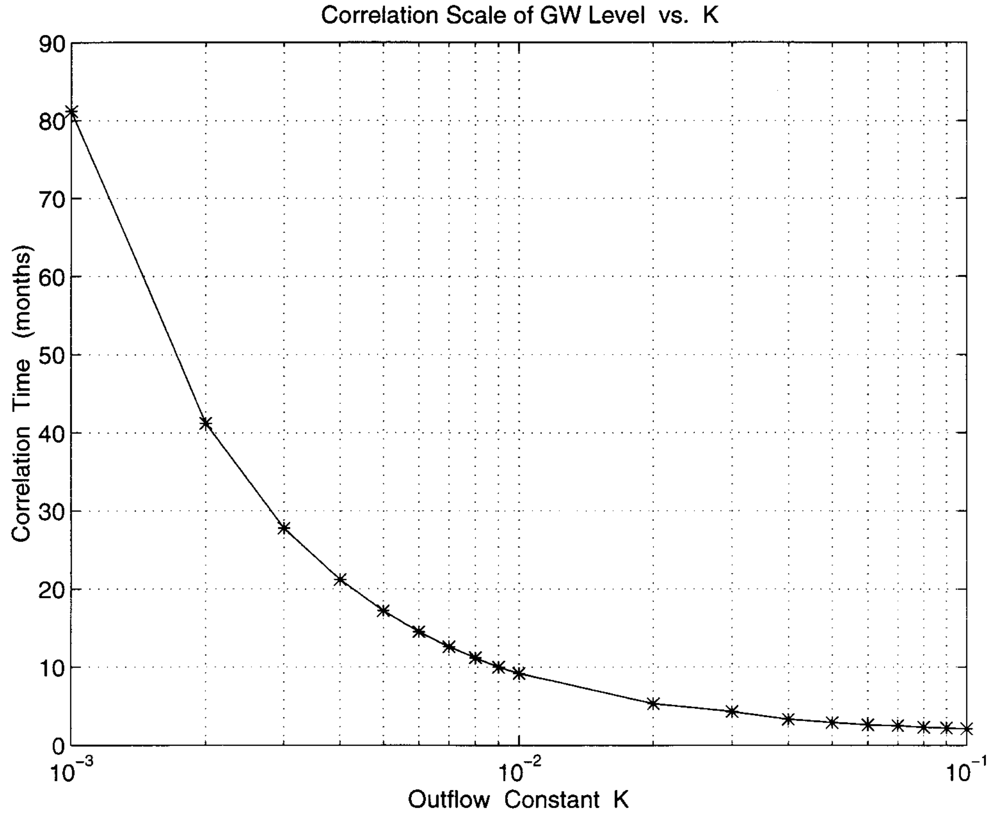

Figure 15. Plot of correlation timescale of groundwater level as a function of outflow constant K. (The correlation time scale of recharge is assumed to be 1 month.)

ship between aquifer water level and base flow. This is particularly true when the ratio of the fluctuations in aquifer water level to the average saturated depth of the aquifer is of relatively small magnitude. Under those conditions variability in h_r would be insignificant. Figure 17 illustrates the relationship between drainage density and aquifer water level. As the level of water rises, the groundwater table intersects with more and more sections of the river network. These sections then provide additional outlets for the aquifer to discharge water into the streams. At the same time, a relatively dense drainage network favors a larger sensitivity of the hydraulic gradients to fluctuations in aquifer water level. The geomorphology of the hill slopes and river network plays a significant role in dictating the nature of the dynamic relationship between aquifer water level and drainage density.

In Illinois the amplitude of the fluctuations in aquifer water level varies between the different regions with a typical magnitude of a few meters. The saturated aquifer thickness varies significantly in the different geological zones. However, a typical scale would be in the order of a few tens of meters (see Appendix A). As a result, the ratio of the fluctuations in aquifer water level to the average saturated depth of the aquifer would be of relatively small magnitude. Hence most likely the dependence of drainage density on aquifer water level is the dominant pathway in shaping the nonlinear relationship between aquifer water level and base flow.

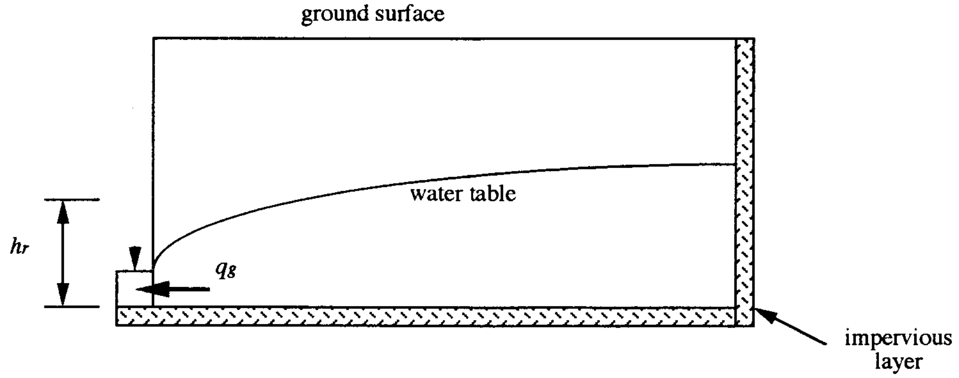

Figure 16. A schematic figure describing the simple groundwater flow configuration that has been assumed in developing equation (9).

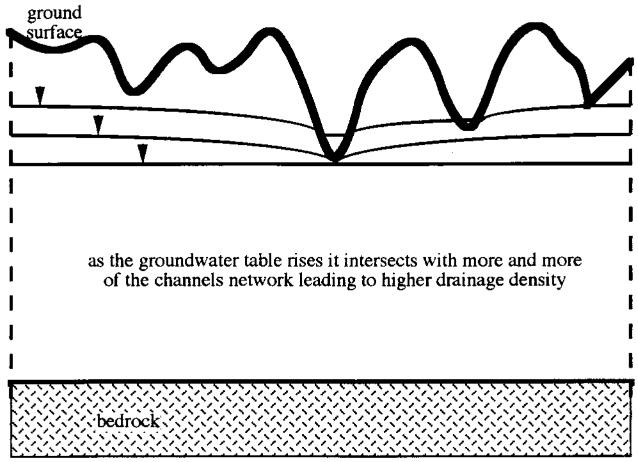

Figure 17. A schematic figure that illustrates the relationship between aquifer water level and drainage density.

7. Conclusions, Potential Implications, and Future Research

1. The climatology of precipitation, evaporation, soil moisture content, groundwater storage, and river flow in Illinois has been estimated. All these variables, except precipitation, exhibit a significant seasonal cycle. The seasonal variability of the incoming solar radiation is the main forcing of the observed seasonality in the regional hydrological cycle of Illinois. Evaporation follows solar radiation closely and leaves a clear signature on the seasonal cycle of soil moisture content. However, because of the slow nature of the water movement within the soil, the seasonal cycle of the groundwater level is similar to that of the soil moisture but lags it by about one month. The seasonal cycle of river flow follows closely that of the groundwater level, which suggests a dominant role for groundwater runoff in the hydrology of Illinois.

2. The anomalies of atmospheric circulation, precipitation, soil moisture content, and groundwater level from their respective averages have been computed to study the transformation of atmospheric circulation anomalies into precipitation anomalies, their subsequent propagation through soil hydrology into anomalies in soil moisture content and groundwater level, and eventually the signature of these anomalies on river flow. While precipitation plays a minor role in shaping the natural variability in the regional hydrological cycle at the seasonal timescale, variability in atmospheric circulations and precipitation are the major factors in shaping the natural variability in the regional hydrological cycle at the interannual timescale.

3. The drought of 1988 and the flood of 1993 have been studied in detail as examples of extreme hydrological anomalies. The 1993 summer flood left a signature in the observed soil moisture content that is more persistent than the corresponding signature of the 1988 summer drought. This difference can be explained as due to the usually wet conditions during fall and winter seasons that are more likely to dissipate a dry soil moisture anomaly than a wet one. However, the 1988 drought anomaly left a signature in the aquifer water level that is more persistent than the corresponding signature for the 1993 summer flood.

4. The discharge to streams from groundwater aquifers is an efficient dissipation mechanism for wet anomalies in aquifer water level, but the nonlinear dependence of the groundwater discharge on aquifer water level may explain why droughts leave a significantly more persistent signature on groundwater hydrology than floods. A theoretical analysis of the aquifer water balance equation, which recognizes this nonlinear relationship, has been performed to elucidate the proposed physical mechanism. The groundwater runoff dissipation mechanism provides a simple explanation for the observed asymmetry in the response of the aquifer to floods and droughts over Illinois. The nonlinear form of the groundwater rating curve reflects to a significant degree the dependence of drainage density on aquifer water level as well as the small changes in transmissivity that results from a typical fluctuation in aquifer water level.

The results of this study may have important implications regarding the impact on regional water resources due to any climate change scenario. Several climate change studies for example, those by *Manabe et al.* [1981], *Rind et al.* [1990], *Wethrald and Manabe* [1995], and *Kattenberg et al.* [1996], have concluded that the evident increase in the concentration of greenhouse gases in Earth's atmosphere is likely to enhance the frequency and severity of drought conditions during summer in midlatitudes. This prediction is consistent with an increase in evaporation that exceeds the corresponding increase in precipitation. The credibility of this prediction is rather limited because of the limitations imposed on the quality of climate model simulation by the inaccuracy in the parameterizations of land surface processes, moisture convection, and clouds, among others. In addition, most of the current climate models that are used in global change studies do not include adequate representations of groundwater aquifers. For this reason such models could not simulate the response of the groundwater aquifers to climate change. The results of the analysis presented in this paper suggest that a change towards a relatively dryer summer climate would get reflected in groundwater levels in an amplified way. A tendency toward dryer summer conditions in terms of precipitation and soil moisture over Illinois is likely to result in relatively more persistent drought conditions in terms of aquifer water levels.

Several research questions have been raised by this study. The observed asymmetry in the patterns of floods and droughts in precipitation over Illinois deserves further investigation into the physical mechanisms responsible for these statistical characteristics. The role of the interactions between temporal fluctuations in aquifer water level and the spatial structure of the basin morphology in dictating the relationship between aquifer water level and drainage density is an open subject for future research. In particular, we speculate that the hypsometric curve, which characterizes the distribution of the basin area among the different elevation zones, is the key geomorphologic measure in shaping the relationship between aquifer water level and drainage density. Finally, further studies that investigate the response of the coupled land-atmosphere system to long-term shifts in climatic conditions are needed in order to determine the role of the mechanisms identified in this paper in shaping the response of the overall system.

Appendix A: Hydrogeology of Illinois

Here we provide a brief description of the hydrogeology of Illinois. For a detailed discussion of the topic, we recommend the earlier studies by the Illinois State Water Survey (ISWS)

[*Leighton et al*., 1948; *Schicht and Walton*, 1961; *Prickett et al*., 1964; *Walton*, 1965; *Changnon et al*., 1988]. The following summary is based upon those studies.

Illinois has a wide diversity of soils which includes almost all the major types of soil found throughout the Midwest. The physiography of Illinois is composed largely of flat glacial prairies and rolling hills [*Changnon et al*., 1988]. Over 90% of Illinois lies mostly within the central low-land physiographic province and is essentially a prairie plain. The relief over most of the state is moderate to slight; large-scale relief features are generally absent.

Large areas in Illinois are covered by unconsolidated deposits left by the glaciers. The glacial deposit commonly ranges from 50 to 200 feet (15–60 m) or more in thickness over Illinois. Glacial till is the principal constituent of the deposit. Till is typically a heterogeneous unsorted mixture of particles ranging from boulders to fine clay. In most places till contains a high percentage of silt and clay such that it has a low permeability and specific yield. Accumulations of wind-blown silt-size materials called loess are associated with glacial deposits. Deposits of loess cover most areas in Illinois to a depth varying from 4 to 25 feet (1–8 m). In addition, these glacial drifts contain extensive deposits of sand and gravel in two zones: near the surface (upper aquifer) and immediately above bedrock (lower aquifer). The upper and lower aquifers exceed 30 feet (9 m) in thickness at many places and are separated by clayey materials (confining bed) commonly exceeding 75 feet (23 m) in thickness.

Moreover, sand and gravel and bedrock aquifers are often deeply buried. The recharge to the aquifers is largely derived from infiltration through thick layers of till that have a low permeability. Groundwater recharge generally is at a maximum during April, and most recharge occurs prior to July. In dry years there is very little recharge from July through November. Vertical leakage is often much less than the infiltration rates into surface deposits, leading to a shallow water table condition in Illinois. In most of the state the water table is near the surface, and shallow ponds, swamps, and poorly drained areas were widespread prior to settlement. Extensive surface and subsurface drainage were necessary to permit agricultural development.

The size and hydraulic properties of the groundwater reservoir in connection with streams determines the quantities of groundwater runoff, which comprises most of the runoff in the extended dry periods. According to an earlier study by ISWS [*Walton*, 1965, p. 17], groundwater runoff in Spring Creek of Illinois ranges from about 75 mm/yr for dry years (i.e., the years of much below normal precipitation) to roughly 250 mm/yr for wet years. The factor most difficult to evaluate, regarding determination of groundwater contribution to the streamflow, is the hydraulic connection between aquifers and streams, and the associated thickness of the contributing aquifers. It is not clear whether the water level fluctuations in any observation well would reflect the hydraulic potential in the corresponding aquifer for providing groundwater discharge to the streams in Illinois. However, as shown in this paper, the close correspondence that has been observed between the water table fluctuations and the magnitude of streamflow in Illinois indicates that the water levels in those shallow wells reflect to a large degree the conditions in the aquifers connected to the streams.

Appendix B: Stochastic Analysis of Equation (8)

Here we describe a stochastic analysis of (8). We will consider S_y, K, H_0, and Q_0 to be constants and H and G to be stationary random functions of time. Thus H and G can be decomposed into a temporal average and a perturbation term ($H = \bar{H} + h$; $G = \bar{G} + g$); then by taking the expected value of (3), the following mean equation is derived:

$$S_y \frac{d\bar{H}}{dt} = \bar{G} - Q_0 - K(\bar{H} - H_0) \tag{B1}$$

By subtracting (B1) from (3), we have

$$S_y \frac{dh}{dt} = g - Kh \tag{B2}$$

which is the perturbation equation governing the fluctuation of input forcing G and output H. The spectral density function of h, $S_{hh}(\omega)$, can be related to that of g, $S_{gg}(\omega)$, through the following relationship:

$$S_{hh}(\omega) = \frac{1}{S_y^2\omega^2 + K^2} S_{gg}(\omega) \tag{B3}$$

Consider the following exponential covariance function [*Gelhar*, 1993] for describing the groundwater recharge G:

$$R_{gg}(\tau) = \sigma_g^2 \exp\left(-\frac{|\tau|}{T_g}\right) \tag{B4}$$

where σ_g^2 and T_g are the variance and correlation timescale of recharge. Another study by the authors focused on the estimation of groundwater recharge in Illinois (P. J.-F. Yeh and E. A. B. Eltahir, unpublished manuscript, 1998) and has shown that the correlation timescale of recharge is about one month. (This value is used as T_g in the plots in Figures 14 and 15). The spectral density function corresponding to the exponential covariance function is

$$S_{gg}(\omega) = \frac{2\sigma_g^2 T_g}{\pi(1 + \omega^2 T_g^2)} \tag{B5}$$

By substituting (B5) into (B3) and then taking the inverse Fourier transform, the covariance function of H can be derived as

$$R_{hh}(\tau) = \sigma_g^2\left[\frac{S_y T_g}{K(S_y^2 - K^2T_g^2)} \exp\left(-\frac{K|\tau|}{S_y}\right) + \frac{T_g^2}{(K^2T_g^2 - S_y^2)} \exp\left(-\frac{|\tau|}{T_g}\right)\right] \tag{B6}$$

The variance of H corresponds to $\tau = 0$

$$\sigma_h^2 = \frac{\sigma_g^2 T_g}{K(KT_g + S_y)} \tag{B7}$$

which implies the larger deviations from the long-term average of groundwater level for a smaller K (i.e., more toward drought conditions).

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